Quantum critical scaling behavior of deconfined spinons

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We perform a renormalization group analysis of some important effective field theoretic models for deconfined spinons. We show that deconfined spinons are critical for an isotropic SU(N) Heisenberg antiferromagnet, if N is large enough. We argue that nonperturbatively this result should persist down to $N = 2$ and provide further evidence for the so called deconfined quantum criticality scenario. Deconfined spinons are also shown to be critical for the case describing a transition between quantum spin nematic and dimerized phases. On the other hand, the deconfined quantum criticality scenario is shown to fail for a class of easy-plane models. For the cases where deconfined quantum criticality occurs, we calculate the critical exponent η for the decay of the two-spin correlation function to first-order in $\epsilon = 4-d$. We also note the scaling relation $\eta = d+2(1-\varphi/\nu)$ connecting the exponent η for the decay to the correlation length exponent ν and the crossover exponent φ.

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The most remarkable incarnation of the Landau-Ginzburg theory of phase transitions is the one embodied by Wilson's renormalization group (RG) [1]. According to this point of view, the Landau-Ginzburg theory is uniquely determined by the effective coupling constants obtained by integrating out high-energy modes. In this way, the large distance scaling behavior of different physical quantities is governed by the fixed points in the space of coupling constants. This is the so called Landau-Ginzburg-Wilson (LGW) paradigm of phase transitions [2]. The LGW paradigm is known to fail in a number of quantum phase transitions. One prominent example is the transition between the Néel state to a valence bond solid (VBS) state in a two-dimensional Mott insulator [3]. This transition features a quantum critical point (QCP), which is at odds with the LGW scenario that would predict a first-order phase transition. The crucial observation in this context is that both phases break symmetries in distinct spaces: the Néel state breaks the SU(2) symmetry of the Hamiltonian, while the paramagnetic VBS state breaks lattice symmetries. A *continuous* such order-order phase transition would not be captured by a LGW-like point of view [4].

For an SU(2) Heisenberg antiferromagnet the spinons z_α are the elementary constituents of the spin orientation field $\mathbf{n}$. We have $n_a = \mathbf{z}^\dagger \sigma_a \mathbf{z}$, $a = 1, 2, 3$, where $\mathbf{z} = (z_1, z_2)$ and σ_a are the Pauli matrices. This is the so called CP^1 representation of the SU(2) spins. There is an inherent *local* gauge invariance in this representation, since $\mathbf{n}$ remains invariant when the spinon fields change by a local phase factor, i.e., $z_\alpha \to e^{i\theta(x)} z_\alpha$. Thus, it seems to be natural to effectively describe a Mott insulator through a gauge theory coupled to "spinon matter". The gauge field here is an *emergent photon*: it is dynamically generated as a consequence of the local gauge invariance of $\mathbf{n}$ in terms of the spinon fields. Note that only expectation values of gauge-invariant operators can be nonzero, in agreement with Elitzur's theorem [5]. The VBS order parameter is also a gauge-invariant expectation value, since it is proportional to $\langle \mathbf{n}_i \cdot \mathbf{n}_j \rangle$, with i and j being nearest neighbor sites in a square lattice. The spinons are confined in both the Néel and VBS phases. Indeed, spinon deconfinement would make $\mathbf{n}$ fall apart leading to a vanishing of both spin and VBS order parameters. A point in the phase diagram where the spinons may deconfine is at a QCP, where both order parameters are supposed to vanish. To actually demonstrate that this happens, is not an easy task. The main argument [2] behind the concept of deconfined quantum criticality (DQC) is a topological one: spinon deconfinement occurs due to a destructive interference mechanism between the instantons and the Berry phase [2]. This mechanism was recently observed numerically [6] for the case of an easy-plane antiferromagnet. However, in this case the instanton cancellation mechanism leads actually to a weak first-order phase transition [6]. In Ref. [7] a first-order phase transition in an easy-plane model of votex loops was also found, but there Berry phase effects were not considered.

One of the most important aspects of DQC is the large value of the critical exponent η for the decay of the correlation function $\mathcal{G}(x) \equiv \langle \mathbf{n}(x) \cdot \mathbf{n}(0) \rangle$ as compared with the value obtained through the LGW approach. This correlation function is highly relevant experimentally. Therefore, it is important to be able to calculate η in a systematic way. The exponent η has been calculated in Monte Carlo simulations for two-dimensional Heisenberg antiferromagnets with instanton suppression [8] and with four-spin interactions [9]. The obtained results are $\eta \approx 0.7$ and $\eta \approx 0.26$, respectively.

One of the main results of the present paper will be the calculation of η in first-order in $\epsilon = 4 - d$, where d is the dimension of space-time. This will be done for two different DQC regimes: (i) the Néel-VBS transition

[2] and (ii) the phase transition between quantum spin nematic and dimerized phases [10].

It was argued in Ref. [2] that for certain isotropic $SU(N)$ symmetric Heisenberg antiferromagnets the quantum critical point is governed by the euclidean Lagrangian

$$\mathcal{L} = \frac{1}{2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2 + \sum_{\alpha=1}^{N} |(\partial_\mu - ie_0 A_\mu)z_\alpha|^2 + r_0 \sum_{\alpha=1}^{N} |z_\alpha|^2 + \frac{u_0}{2}\left(\sum_{\alpha=1}^{N} |z_\alpha|^2\right)^2, \quad (1)$$

where the parameter r_0 is tuned in such a way that the system is at its critical point. The above Lagrangian corresponds to an Abelian Higgs model in euclidean space with an $O(2N)$ global symmetry. It can also be thought of as the free energy of a Ginzburg-Landau (GL) model with N complex order parameter fields. In this case the upper critical dimension is four. Thus, the RG analysis should be made in $d = 4 - \epsilon$ dimensions. We define dimensionless couplings $g = \mu^{-\epsilon}u_r$ and $f = \mu^{-\epsilon}e_r^2$, where u_r and e_r are the renormalized counterparts of u_0 and e_0, respectively. The RG β functions $\beta_f \equiv \mu\partial f/\partial\mu$ and $\beta_g \equiv \mu\partial g/\partial\mu$ are straightforwardly obtained employing standard techniques [11]:

$$\beta_f = -\epsilon f + \frac{N}{3}f^2, \quad (2)$$

$$\beta_g = -\epsilon g - 6fg + (N+4)g^2 + 6f^2. \quad (3)$$

The above RG equations are well known in the context of the GL model [12]. An infrared stable fixed point with $f \neq 0$ is found only for N large enough, namely, $N > 182.9$. Unless N is greater than this value, no second-order phase transition is predicted by this RG analysis. In the past this result led to the conclusion [12] that thermal fluctuations turn the phase transition in a superconductor into a first-order one, since there the actual number of components is $N = 1$. It did not take too long to realize that this result is incorrect [13, 14]. Actually the large, but finite, N result reflects the strong-coupling features of the $N = 1$ theory, which cannot be captured by the RG analysis in $d = 4 - \epsilon$ dimensions. *This does not mean that the first-order transition cannot occur.* It turns out that the complete phase diagram features a tricritical point [14] at a value of the Ginzburg parameter $\kappa = u/\sqrt{2}e$ smaller than the mean-field GL value separating the type I from the type II regimes, i.e., $\kappa = 1/\sqrt{2}$. Earlier calculations based on duality arguments [14] give the value $\kappa_t \approx 0.8/\sqrt{2}$, a result recently confirmed by large scale Monte Carlo simulations [15]. The weak-coupling regime at low N captured by the RG functions (2) and (3) corresponds to the one in which $\kappa \ll \kappa_t$. An RG analysis in fixed dimension $d = 3$ [16], though less well controlled than the one near four dimensions, indicates that the critical value of N can be drastically reduced if a resummed higher order calculation is performed [17]. Other interesting effects arise as the number of components gets large enough for the case where the gauge field is compact. For example, there is a recent numerical evidence for a Coulomb-like phase in a compact abelian gauge theory coupled to multiflavor CP^1 fields [18].

We can make an analysis of deconfined quantum criticality that parallels the case of superconductors in the neighborhood of the critical temperature. In contrast to the superconductor, in this case the physical value of the parameter N is given by $N = 2$. Hence, we have two classes of (meron) vortices [2, 19]. The above discussion in the context of superconductors shows that in principle we can also have *deconfined quantum tricriticality*.

The correlation function $\mathcal{G}(x) = \langle \mathbf{n}(x) \cdot \mathbf{n}(0)\rangle$ in the CP^{N-1} representation reads

$$\mathcal{G}(x) = 2\langle \mathbf{z}^*(x)\cdot\mathbf{z}(0)\ \mathbf{z}(x)\cdot\mathbf{z}^*(0)\rangle - \frac{2}{N}\langle |\mathbf{z}(x)|^2|\mathbf{z}(0)|^2\rangle. \quad (4)$$

At the deconfined QCP this correlation function scales as $\mathcal{G}(x) \sim 1/|x|^{d-2+\eta}$ [20]. In order to compute η, let us analyse the scaling behavior of the two four-spinon correlation functions in Eq. (4).

The scaling behavior of $\langle |\mathbf{z}(x)|^2|\mathbf{z}(0)|^2\rangle$ is obtained by considering the scaling dimension of the operator $|\mathbf{z}(x)|^2$. This is easily obtained by performing derivatives with respect to r_0 of the logarithm of the functional integral and doing dimensional analysis. The result is a scaling behavior of the form $\langle |\mathbf{z}(x)|^2|\mathbf{z}(0)|^2\rangle \sim 1/|x|^{d-2+\eta_4}$, where $\eta_4 = d + 2(1 - 1/\nu)$, with ν being the correlation length exponent. This leads to a vanishing of those correlations in momentum space as $p \to 0$, except for the mean-field case where $\eta_4 = d - 2$. Indeed, beyond mean-field theory we have $\nu > 2/d$ and thus it is clear that $\eta_4 > d - 2$ when the fluctuations are included. This result is important because it legitimates the softening of the CP^{N-1} constraint $|\mathbf{z}|^2 = 1$. In the critical regime we can simply neglect the second term in Eq. (4).

Let us consider now the scaling behavior of $\langle \mathbf{z}^*(x) \cdot \mathbf{z}(0)\ \mathbf{z}(x)\cdot\mathbf{z}^*(0)\rangle$. This correlation function is associated with a mass anisotropy term, which is obviously not generated by quantum fluctuations in a SU(N) theory like the one in Eq. (1). However, we can consider it as a source term and compute the so called crossover exponent φ [21]. The exponent η is then obtained by replacing $1/\nu$ in the expression for η_4 by φ/ν. Therefore, $\langle \mathbf{z}^*(x)\cdot\mathbf{z}(0)\ \mathbf{z}(x)\cdot\mathbf{z}^*(0)\rangle \sim 1/|x|^{d-2+\eta}$, where $\eta = d + 2(1 - \varphi/\nu)$. This result is obtained as follows.

The anomalous dimensions of all quadratic operators, leading to mass anisotropy or not, can be derived from a "matrix exponent" $\eta^{(2)}_{\alpha\beta,\gamma\delta} =$

$\lim_{\mu\to 0}\mu\partial\ln[Z^{(2)}_{\alpha\beta,\mu\nu}(Z^{-1/2})_{\mu\gamma}(Z^{-1/2})_{\nu\delta}]/\partial\mu$ (here a summation over repeated greek indices is implied), where $Z^{(2)}_{\alpha\beta,\mu\nu}$ is the renormalization associated to the insertion of a quadratic operator and $Z_{\alpha\beta}$ the spinon wave function renormalization [21]. From the eigenvalues of this matrix exponent we can determine both ν and φ or, equivalenty, η_4 and η. These eigenvalues are the anomalous dimensions of the composite operators $|\mathbf{z}|^2$ and $z^*_\alpha z_\beta$, with $\alpha\neq\beta$. We will call these anomalous dimensions η_2 and η_2', respectively. Now it is straightforward to use dimensional analysis to obtain that $\eta_4=d-2-2\eta_2=d+2(1-1/\nu)$ and $\eta=d-2-2\eta_2'=d+2(1-\varphi/\nu)$.

For the DQC regime described by Eq. (1) we have at one-loop order

$$\eta^{(2)}_{\alpha\beta,\gamma\delta}=-NP_{\alpha\beta,\gamma\delta}g_*+(3f_*-g_*)I_{\alpha\beta,\gamma\delta},\tag{5}$$

where $I_{\alpha\beta,\gamma\delta}=(\delta_{\alpha\gamma}\delta_{\beta\delta}+\delta_{\alpha\delta}\delta_{\beta\gamma})/2$, $P_{\alpha\beta,\gamma\delta}=\delta_{\alpha\beta}\delta_{\gamma\delta}/N$, and g_* and f_* are the infrared stable fixed points associated with the β functions (3) and (2). The eigenvalue η_2 corresponding to the eigenvector $\delta_{\gamma\delta}$ determines the critical exponent ν as $1/\nu=2+\eta_2$. The second eigenvalue, η_2' determines φ through $\eta_2'=\varphi/\nu-2$. Explicitly, we have $\eta_2=-(N+1)g_*+3f_*$ and $\eta_2'=-g_*+3f_*$. Therefore, we obtain to order ϵ and for $N>182.9$ the result

$$\eta=2-\left(1+\frac{18}{N}\right)\epsilon+2g_*,\tag{6}$$

where $g_*=(18+N+\sqrt{N^2-180N-540})\epsilon/[2N(N+4)]$. Using the lowest value of N for which the stable fixed point exists (i.e., $N=183$), we obtain after setting $\epsilon=1$ the result $\eta=609/671\approx 0.9076$. The LGW result to order ϵ would give $\eta=0$ and a small correction to order ϵ^2. Note that the local gauge invariance is essential in order to get a value smaller than one for η. If we consider the model without any gauge coupling, we obtain $\eta=2-[1-2/(N+4)]\epsilon$, which is an expression valid for all values of N, since in this case the perturbative fixed point exists even for $N=1$. This leads for $N=183$ and $\epsilon=1$ to the result $\eta=189/187>1$.

Now we consider the scaling behavior of the spin $S=1$ Hamiltonian [10, 22]

$$H=\sum_{\langle i,j\rangle}\left[J\,\mathbf{S}_i\cdot\mathbf{S}_j-K(\mathbf{S}_i\cdot\mathbf{S}_j)^2\right],\tag{7}$$

where both nearest-neighbor couplings J and K are positive. This Hamiltonian describes the phase transition between a quantum spin nematic phase and a dimerized phase. Recent numerical results indicate that this model exhibits a second-order phase transition if the ratio K/J is large enough. As pointed out in Ref. [10], the LGW paradigm would in this case predict a first-order phase transition, at odds with the results observed numerically [22]. Here we will show using the RG that in this model a second-order phase transition occurs for large enough N. The field theory of the above model was derived recently [10] and is given by the Lagrangian

$$\begin{aligned}\mathcal{L}&=\frac{1}{2}(\epsilon_{\mu\nu\lambda}\partial_\nu A_\lambda)^2+|(\partial_\mu-ie_0A_\mu)\mathbf{D}|^2+r_0|\mathbf{D}|^2\\&+\frac{u_0+v_0}{2}(|\mathbf{D}|^2)^2-\frac{v_0}{2}(\mathbf{D})^2(\mathbf{D}^*)^2,\end{aligned}\tag{8}$$

where $v_0>0$ and $\mathbf{D}$ is a complex vector with three components.

To see how the second-order transition emerges, let us write $D_i=(\varphi_i+i\psi_i)/\sqrt{2}$, with $i=1,2,3$. The local interaction between the scalar fields become

$$\mathcal{L}_{\rm int}=\frac{u_0}{8}(\boldsymbol{\varphi}^2+\boldsymbol{\psi}^2)^2-\frac{v_0}{2}(\boldsymbol{\varphi}\cdot\boldsymbol{\psi})^2.\tag{9}$$

The above equation features an interaction reminiscent of certain classical models for frustated magnetism [23]. In order to perform the RG analysis, we will consider a generalization of the model such that $\boldsymbol{\varphi}$ and $\boldsymbol{\psi}$ have each N components, with the physically relevant case corresponding to $N=3$. The β function for the gauge coupling is given once more by Eq. (2). By introducing dimensionless couplings $g=\mu^{-\epsilon}u$ and $h=\mu^{-\epsilon}v$ we obtain the one-loop β functions:

$$\beta_g=-\epsilon g-6fg+(N+4)g^2+2h^2-2gh+6f^2,\tag{10}$$

$$\beta_h=-\epsilon h-6fh-(N+2)h^2+6gh.\tag{11}$$

It is useful to analyse first the case where $f=0$. The physically meaningful case corresponds to fixed points where $h\geq 0$. The relevant fixed point in this case has coordinates $g_*=2\epsilon/(N^2+8)$ and $h_*=(2-N)\epsilon/(N^2+8)$. This fixed point is stable only for $N=3$, but then we would have $h_*<0$, which is incompatible with the physical constraints of the model [10].

Remarkably, for $f_*=3\epsilon/N$ a stable fixed point is found for $N>232.98$:

$$g_*=\frac{360-12N+2N^2+N^3+(N+2)\sqrt{\Delta}}{2N(64+8N+8N^2+N^3)}\,\epsilon,\tag{12}$$

$$h_*=\frac{3\sqrt{\Delta}-36-104N-21N^2-N^3}{N(64+8N+8N^2+N^3)}\,\epsilon,\tag{13}$$

where $\Delta=N^4-224N^3-2072N^2-4608N-22896$. Once more, just like in the isotropic case, we interpret the existence of stable fixed points at large values of N as a strong evidence of deconfined quantum criticality. The exponent η is calculated similarly as before, giving the result $\eta\approx 0.927$ for $N=233$ and $\epsilon=1$.

Next, we consider an easy-plane version of the model (1). This amounts to adding an interaction term of the

form $v_0(|z_1|^2 - |z_2|^2)^2/2$. Previous results on the easy-plane model [2, 8] indicated that a second-order phase transition would occur. This conclusion was based on the analysis of the *deep* easy-plane limit of the model. This regime corresponds to a large v_0 such that $|z_1|^2 \approx |z_2|^2$. However, recent Monte Carlo simulations [6, 7] performed in this regime showed that the transition is actually (weakly) first-order.

In order to facilitate the RG calculations it is convenient to write the complete local interaction between the spinons in the following form:

$$\mathcal{L}_{\rm int} = \frac{\bar{u}_0}{2}(|z_1|^4 + |z_2|^4) + w_0|z_1|^2|z_2|^2, \qquad (14)$$

where $\bar{u}_0 = u_0 + v_0$ and $w_0 = u_0 - v_0$. Let us introduce the renormalized dimensionless couplings $g = \bar{u}\mu^{-\varepsilon}$ and $h = w\mu^{-\varepsilon}$, where $\bar{u}$ and w are the renormalized counterparts of $\bar{u}_0$ and w_0, respectively. In order to have the same total number of complex components as before, we will consider $N/2$ components of z_1 and z_2, with N even. By this we mean a rewriting of the interaction, such that the system has a $O(N) \times O(N)$ symmetry. The one-loop β function for the gauge coupling is the same as before. The other β functions are

$$\beta_{\bar{g}} = -\epsilon g - 6gf + \frac{N+8}{2}g^2 + \frac{N}{2}h^2 + 6f^2, \qquad (15)$$

$$\beta_h = -\epsilon h - 6hf + (N+2)gh + 2h^2 + 6f^2. \qquad (16)$$

It is instructive to consider first the model for vanishing gauge coupling ($f = 0$). In this case besides the Gaussian ($g_* = h_* = 0$) and Heisenberg [$g_* = 2\epsilon/(N+8)$ and $h_* = 0$] fixed points, we have the fixed points $g_1 = h_1 = \epsilon/(N+4)$, and (g_2, h_2) with $g_2 = N\epsilon/(N^2+8)$ and $h_2 = (4-N)\epsilon/(N^2+8)$. From these only the fixed point (g_2, h_2) is infrared stable, provided $N = 3$. Note that for $N = 4$ we have the realization of the deep easy-plane limit, since the effective interaction multiplying $|z_1|^2|z_2|^2$ vanishes, although the bare coupling $w_0 \neq 0$. However, such a case does not corresponds to a stable fixed point. Note that for $N = 2$ the fixed point (g_2, h_2) is $O(4)$ symmetric and coincides with (g_1, h_1).

Note that the gauge field fluctuations, which in this problem are essential, generate a $|z_1|^2|z_2|^2$ term. Thus, we have to keep $w_0 \neq 0$ and look in the RG treatment for the stability of fixed points with $h = 0$. It turns out that *for all values of N, no fixed points with $h_* = 0$ and $f_* = 3\epsilon/N$ are found*. Therefore, no second-order phase transition takes place in this case. This is a very significant result, since as we have discussed, the existence of a critical value of N above which the transition becomes second-order reflects the actual behavior at lower values of N. The complete absence of fixed points for all N provides a solid theoretical explanation for the numerical results of Refs. [7] and [6]. Fixed points with $h_* \neq 0$ exist for large enough N, but none of them are stable. Therefore, there is no deconfined quantum criticality associated to the model (14).

Summarizing, we have considered three models for deconfined spinons. From the three models considered, only the one associated with an easy-plane antiferromagnet does not exhibit any second-order phase transition, in agreement with the numerical results of Refs. [6] and [7]. Deconfined spinons were shown to govern a second-order phase transition for both the isotropic $SU(N)$ antiferromagnet and quantum spin nematic systems. In both cases we have computed the critical exponent η using the scaling relation $\eta = d + 2(1 - \varphi/\nu)$ in terms of the crossover exponent φ and the correlation length exponent ν. Knowledge of the exponent φ is very important in the study of crossover behavior and stability of frustrated systems.

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